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RADIANT HEAT TRANSFER FROM STORAGE CASKS TO THE ENVIRONMENT*

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ABSTRACT

A spent fuel storage cask must efficiently transfer the heat released by the fuel assemblies through the cask walls to the environment. This heat must be transferred through passive means, limiting the energy transfer mechanisms from the cask to natural convection and radiation heat transfer. Natural convection is essentially independent of the characteristics of the array of casks, provided there is space between casks to permit a convection loop. Radiation heat transfer, however, depends on the geometric arrangement of the array of casks because the peripheral casks will shadow the interior casks and restrict radiant heat transfer from all casks to the environment.

The shadowing of one cask by its neighbors is determined by a view factor that represents the fraction of radiant energy that leaves the surface of a cask and reaches the environment. This paper addresses the evaluation of the view factor between a centrally located spent fuel storage cask and the environment. By combining analytic expressions for the view factor of (1) infinitely long cylinders and (2) finite cylinders with a length-to-diameter ratio of 2 to represent spent fuel storage casks, the view factor can be evaluated for any practical array of spent fuel storage casks.

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1.0 INTRODUCTION

A spent fuel storage cask must efficiently transfer the heat released by the fuel assemblies contained in the cask through the cask walls to the environment. This heat must be transferred by passive means, limiting the energy transfer mechanisms to natural convection and radiation heat transfer. Natural convection is essentially independent of the characteristics of the array of casks, provided there is space between casks to permit a convection loop. However, radiation heat transfer depends on the geometric arrangement of the array of casks since the peripheral casks will shadow the interior casks and restrict radiant heat transfer from all casks to the environment. For this discussion, the possibility of radiant heat transfer between adjacent casks within the array will be neglected for conservatism and because immediately adjacent casks would be close to the same temperature. Thus, only direct radiant heat transfer between any cask and the environment will be addressed.

The basic relationship for radiant heat transfer between any cask and the environment is (Seigel 1972):

$$q_{s-e} = \frac{\sigma A_s\left(T_s^4 - T_e^4\right)}{\frac{1}{F_{s-e}} + \left(\frac{1}{\varepsilon_s} - 1\right) + \frac{A_s}{A_e}\left(\frac{1}{\varepsilon_e} - 1\right)},$$

where:

the subscript **e** represents temperatures, areas or emissivities of the environment,

the subscript **s** represents temperatures, areas or emissivities of the surface of the storage cask,

q_{s-e} is the net heat transfer from the surface **s** to the environment **e**,

A is the surface area of the surface of the cask or the environment,

σ is the Stefan-Boltzmann constant (σ = 5.66961×10^{-8} W/m^2 K^4),

T is the absolute temperature of the surface or the environment,

F_{s-e} is the view factor representing the fraction of radiant energy leaving the surface that reaches the environment

ε is the emissivity of the surface or the environment.

The shadowing of one cask by its neighbors is determined by the view factor (F_{s-e}) that represents the fraction of radiant energy that leaves the surface of a cask and reaches the environment. For this discussion, it will be assumed that the storage casks are arranged in a uniform, square array. To determine the direct radiant heat transfer between a cask and the environment, it is necessary to characterize the effective view factor for a given cask, with a surface **s**, as:

$$F_{s-e} = 1 - \sum_{i=1}^{N} F_{s-i},$$

where:

F_{s-e} is the view factor representing the fraction of radiant energy leaving the surface that reaches the environment,

F_{s-i} is the view factor representing the fraction of radiant energy leaving the surface that reaches all other casks in the array,

i is the index that spans all other casks in the array, and

N is the total number of casks in the array.

For an individual spent fuel storage cask, the ratio of the effective area of the surrounding environment to that of the cask is very large ($A_s/A_e \rightarrow 0$) so the expression for the radiant energy exchange reduces to:

$$q_{s-e} = \frac{\sigma A_s\left(T_s^4 - T_e^4\right)}{\frac{1}{F_{s-e}} + \left(\frac{1}{\varepsilon_s} - 1\right)}.$$

Thus, the evaluation of the radiant heat transfer is dependent upon the surface emissivity (ε_s), (a material and surface property) and the view factor (F_{s-e}) (dependent only upon the geometry of the spent fuel storage cask). The view factor between a cask in an array of similar casks and the environment is less than unity because the intervening casks obstruct the reference cask.

A typical array of spent fuel storage casks is presented in Figure 1. The casks of interest in this study are identified by numbers 1 to 6. To indicate typical symmetry, additional casks that occupy position 2 are indicated. It will be assumed that the radiant interchange between cask 1 and all casks identified as cask 6 is negligible. For widely spaced arrays, this view factor could be non-negligible; however, typical arrays of spent fuel storage casks have a pitch-to-diameter ratio (P/D) of about 2, resulting in very small values for the view factor between casks 1 and 6. This situation will be verified later in the discussion for an array of casks with a P/D of 2.

For this analysis, the spent fuel storage casks have been idealized as right, circular cylinders. It is further assumed that each of the spent fuel storage casks has a uniform external surface temperature. Ignoring the longitudinal changes in diameter of a cask and ignoring the temperature variations within each spent fuel storage cask provides an upper bound on the radiation heat transfer.

There are three parameters that characterize the array of spent fuel storage casks (or cylinders): cask diameter, cask length, and spacing between casks on the storage pad. To simplify the study of these three parameters, the relationships between the view factor and the P/D will be studied for very long (essentially infinite length) cylinders. Then the dependence of the view factor on length-to-diameter ratio will be studied for a fixed spacing (P/D=2) on the storage pad. The combination of these two analyses will permit the evaluation of the view factor for arbitrary combinations of the three parameters without requiring calculations for every case.

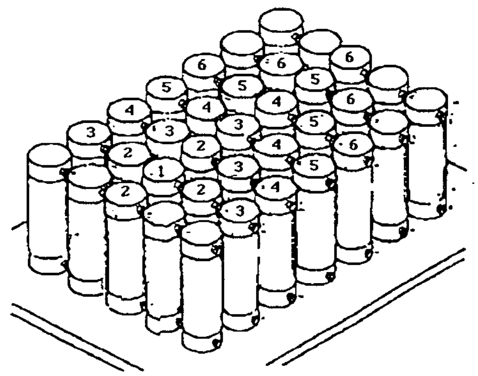

FIGURE 1. ARRAY OF STORAGE CASKS

2. VIEW FACTORS FOR INFINITELY LONG CYLINDERS IN A SQUARE ARRAY

The view factors for infinitely long cylinders in an array can be determined by referring to Hottel's String Rule (Hottel 1954; Seigel 1972), which states that the view factor between two infinitely long bodies is proportional to the difference between the lengths of crossed strings that touch the extremities of the bodies and the lengths of strings that touch the extremities of the bodies without crossing. This is depicted in Figure 2 and the following equation:

$$F_{1-2} = \frac{\overline{ae} + \overline{fi} - \overline{ai} - \overline{fe}}{2\pi D} .$$

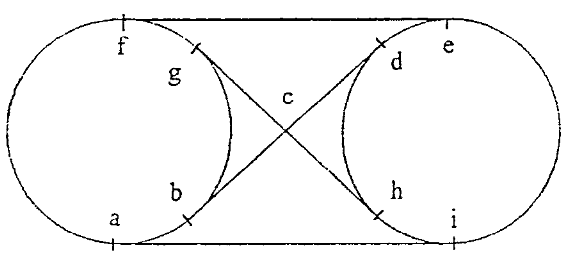

FIGURE 2. HOTTEL'S STRING RULE

For two adjacent right circular cylinders, the resulting expression for the view factor is (Seigel 1972):

$$F_{1-2} = \frac{1}{\pi}\left(\sqrt{\left(\frac{P}{D}\right)^2 - 1} - \frac{P}{D} + \sin^{-1}\left(\frac{D}{P}\right)\right).$$

The only variable in this expression is the P/D of the cylinders. This expression applies to the view factor between cylinder 1 and any cylinder labeled 2 in Figure 1. If the P/D is unity (i.e., the cylinders are touching) this reduces to $(1/2 - 1/\pi)$, approximately 0.18. This is less than 0.25, indicating some radiant heat transfer to cylinder 3.

Figure 3 shows a top view of four adjacent cylinders, illustrating the geometry for the view factor between cylinders 1 and 3. When the P/D is greater than $\sqrt{2}$, the adjacent cylinders do not shadow the receiving cylinder, so the expression for the view factor is the same as that for the view factor for adjacent cylinders (F_{1-2}), except that the separation between cylinders is increased to $\sqrt{2}\,P$. Alternately stated, when the P/D is greater than $\sqrt{2}$, the strings that connect the extremities of the cylinders do not touch the adjacent cylinders. In Figure 3, the P/D ratio is slightly less than $\sqrt{2}$, causing some shadowing of cask 3 by both cylinders labeled 2.

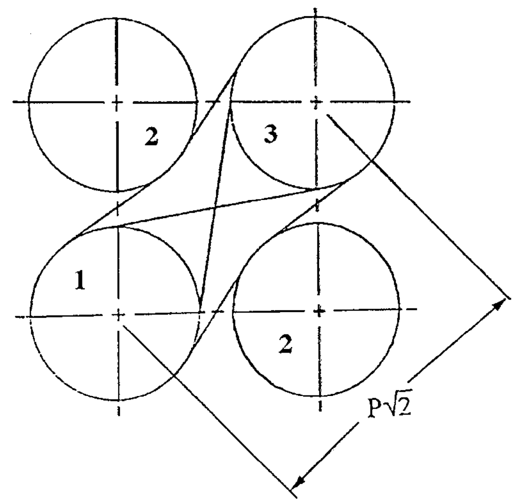

FIGURE 3. TOP VIEW OF FOUR CYLINDERS

The expression for the contact angle between the strings connecting the extremities of the cylinders and the intervening cylinders is $(\pi/4 - \cos^{-1}(D/P))$ which reduces to zero when the P/D is equal to $\sqrt{2}$; the contact angle is negative (i.e., no contact) when the P/D is greater than $\sqrt{2}$.

If the P/D is less than $\sqrt{2}$, as shown in Figure 3, the view factor that characterizes the radiant heat transfer between cylinders 1 and 3 is:

$$F_{1-3} =$$
$$\frac{1}{\pi}\left(\sqrt{2\left(\frac{P}{D}\right)^2 - 1} - 2\sqrt{\left(\frac{P}{D}\right)^2 - 1} + \sin^{-1}\left(\frac{D}{\sqrt{2}P}\right) - 2\left(\frac{\pi}{4} - \cos^{-1}\left(\frac{D}{P}\right)\right)\right),$$

and if the P/D is greater than $\sqrt{2}$, then:

$$F_{1-3} = \frac{1}{\pi}\left(\sqrt{2\left(\frac{P}{D}\right)^2 - 1} - \sqrt{2}\,\frac{P}{D} + \sin^{-1}\left(\frac{D}{\sqrt{2}P}\right)\right).$$

Figure 4 shows a top view of six adjacent cylinders, the geometry that represents radiant heat transfer (i.e., the view factor) between cylinders 1 and 4.

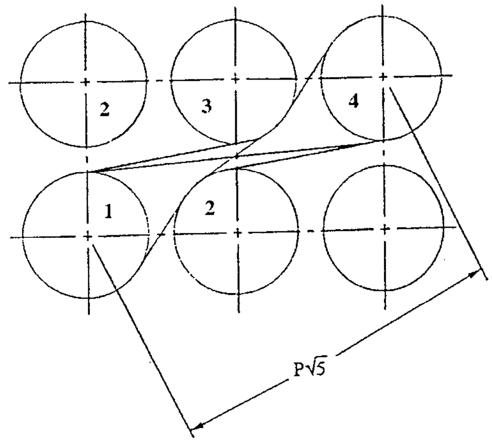

FIGURE 4. TOP VIEW OF SIX CYLINDERS

Three ranges must be considered for this view factor, each with a different expression that results from various combinations of cylinders intersecting the region between cylinders 1 and 4. The first range spans P/D between 1 and $\sqrt{2}$ as shown in Figure 4. When P/D is less than $\sqrt{2}$, one of the crossed strings does not intersect either of the adjacent cylinders, but the remaining crossed string intersects both of the adjacent cylinders. The second range spans P/D from $\sqrt{2}$ to 2.23605. In this range, both of the crossed strings are free of interference from the adjacent cylinders. However, both of the uncrossed strings intersect the adjacent cylinders. The following view factors characterize radiant heat transfer in two ranges.

$$F_{1-4} = \frac{1}{\pi}\left(\frac{1}{2}\sqrt{5\left(\frac{P}{D}\right)^2 - 1} - \sqrt{2\left(\frac{P}{D}\right)^2 - 1} + \frac{1}{2}\sqrt{\left(\frac{P}{D}\right)^2 - 1} + \frac{1}{2}\sin^{-1}\left(\frac{D}{\sqrt{5}P}\right)\right)$$
$$- \frac{1}{\pi}\left(\left(\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{D}{P}\right) - \cos^{-1}\left(\frac{D}{\sqrt{2}P}\right)\right)\right).$$

THIS VIEW FACTOR CHARACTERIZES RADIANT HEAT TRANSFER BETWEEN CYLINDERS 1 AND 4 IN THE RANGE P/D LESS THAN $\sqrt{2}$

$$F_{1-4} = \frac{1}{\pi}\left(\sqrt{5\left(\frac{P}{D}\right)^2 - 1} - \sqrt{2\left(\frac{P}{D}\right)^2 - 1} - \sqrt{\left(\frac{P}{D}\right)^2 - 1} + \sin^{-1}\left(\frac{D}{\sqrt{5}P}\right)\right)$$
$$- \frac{1}{\pi}\left(\left(\frac{3\pi}{4} - \cos^{-1}\left(\frac{D}{P}\right) - \cos^{-1}\left(\frac{D}{\sqrt{2}P}\right)\right)\right).$$

THIS VIEW FACTOR CHARACTERIZES RADIANT HEAT TRANSFER BETWEEN CYLINDERS 1 AND 4 IN THE RANGE P/D FROM $\sqrt{2}$ TO 2.23605

If P/D exceeds 2.23605, both the crossed and uncrossed strings are free of interference with the adjacent cylinders. The expression for the contact angle between the uncrossed strings and the adjacent cylinders is ($3\pi/4 - \cos^{-1}(D/P) - \cos^{-1}(D/\sqrt{2}P)$). This contact angle approaches zero at a P/D of 2.23605 and is negative (i.e., no contact) for P/D greater than 2.23605. The view factor that characterizes the radiant heat transfer between cylinders 1 and 4 in this range is:

$$F_{1-4} = \frac{1}{\pi}\left(\sqrt{5\left(\frac{P}{D}\right)^2 - 1} - \sqrt{5}\,\frac{P}{D} + \sin^{-1}\left(\frac{D}{\sqrt{5}P}\right)\right).$$

Figure 5 shows a top view of eight adjacent cylinders, the geometry that characterizes radiant heat transfer (i.e., the view factor) between cylinders 1 and 5.

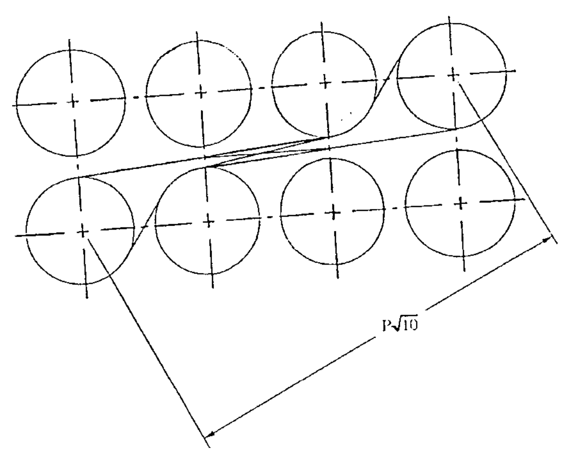

FIGURE 5. TOP VIEW OF EIGHT CYLINDERS

As with the view factor between cylinders 1 and 4, there are three ranges with different expressions for the view factor between cylinders 1 and 5. The first range spans P/D from 1 to 2.23605, depicted in Figure 5. When the P/D is less than 2.23605, one of the crossed strings and both of the uncrossed strings intersect with the adjacent cylinders. The view factor characterizing the radiant heat transfer between cylinders 1 and 5 in this range is shown below:

$$
\begin{aligned}
F_{1-5} = {} & \frac{1}{2\pi}\left(\sqrt{10\left(\frac{P}{D}\right)^2-1}-2\sqrt{5\left(\frac{P}{D}\right)^2-1}+\sqrt{2\left(\frac{P}{D}\right)^2-1}+3\sqrt{\left(\frac{P}{D}\right)^2-1}\right)\\
& +\frac{1}{2\pi}\left(\sin^{-1}\left(\frac{D}{\sqrt{10}P}\right)-3\cos^{-1}\left(\frac{D}{P}\right)-\cos^{-1}\left(\frac{D}{\sqrt{2}P}\right)-\cos^{-1}\left(\frac{D}{\sqrt{5}P}\right)\right)\\
& +\frac{1}{2\pi}\left(-\frac{3}{2}\tan^{-1}\left(\frac{1}{3}\right)-\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)-\frac{1}{2}\tan^{-1}(3)+\frac{1}{2}\tan^{-1}(2)-\frac{9}{4}\pi\right).
\end{aligned}
$$

The second range spans P/D from 2.23605 to 3.1622777. In this range, both of the crossed strings are free of interference from the adjacent cylinders. However, both of the uncrossed strings intersect the adjacent cylinders. The expression for the contact angle between one of the crossed strings and an adjacent cylinder is ($3\pi/4 - \cos^{-1}(D/P) - \cos^{-1}(D/\sqrt{2}P)$). This contact angle approaches zero at a P/D of 2.23605 and is negative (i.e., no contact) for P/D greater than 2.23605. The view factor characterizing the radiant heat transfer between cylinders 1 and 5 in this range is:

$$
\begin{aligned}
F_{1-5} = {} & \frac{1}{\pi}\left(\sqrt{5\left(\frac{P}{D}\right)^2-1}-\sqrt{2\left(\frac{P}{D}\right)^2-1}-\sqrt{10\left(\frac{P}{D}\right)^2-1}\right)\\
& +\frac{1}{\pi}\left(\sin^{-1}\left(\frac{D}{\sqrt{5}P}\right)-\left(\frac{3\pi}{4}-\cos^{-1}\left(\frac{D}{P}\right)-\cos^{-1}\left(\frac{D}{\sqrt{2}P}\right)\right)\right)
\end{aligned}
$$

If the P/D is greater than 3.1622777, both the crossed and uncrossed strings are free of interference from the adjacent cylinders. The expression for the contact angle between the uncrossed strings and the adjacent cylinders is $\{\pi/2-[\tan^{-1}(3)-\tan^{-1}(2)]-\cos^{-1}(D/P\sqrt{5})\}$. This contact angle approaches zero at a P/D of 3.1622777 and is negative (i.e., no contact) for P/D greater than 3.1622777. The view factor characterizing the radiant heat transfer between cylinders 1 and 5 in this range is:

$$
F_{1-5} = \frac{1}{\pi}\left(\sqrt{10\left(\frac{P}{D}\right)^2-1}-\sqrt{10}\frac{P}{D}+\sin^{-1}\left(\frac{D}{\sqrt{10}P}\right)\right).
$$

The view factors between cylinders for the above expressions have been evaluated as a function of the P/D and are plotted in Figure 6.

The view factors that characterize the transfer of radiant energy from an infinite cylinder in an array of identical cylinders to the environment are combinations of the above view factors. Assuming the array is composed of long rows of cylinders where the number of cylinders in a row is much larger than the number of rows in the array, the minimum radiant heat transfer from a storage cask located in the central row of an array of casks to the environment is given for:

One row of cylinders:

$$F_{1\text{-}c} = 1 - 2F_{1\text{-}2}$$

Two rows of cylinders:

$$F_{1\text{-}c} = 1 - 3F_{1\text{-}2} - 2F_{1\text{-}3} - 2F_{1\text{-}4} - 2F_{1\text{-}5}$$

Three rows of cylinders:

$$F_{1\text{-}c} = 1 - 4F_{1\text{-}2} - 4F_{1\text{-}3} - 4F_{1\text{-}4} - 4F_{1\text{-}5}$$

Four rows of cylinders:

$$F_{1\text{-}c} = 1 - 4F_{1\text{-}2} - 4F_{1\text{-}3} - 6F_{1\text{-}4} - 4F_{1\text{-}5}$$

Five rows of cylinders:

$$F_{1\text{-}c} = 1 - 4F_{1\text{-}2} - 4F_{1\text{-}3} - 8F_{1\text{-}4} - 4F_{1\text{-}5}$$

The view factors between infinite cylinders and the relationships for the view factors between the limiting cylinder (storage cask) and the environment are evaluated as a function of the P/D and shown in Figure 7.

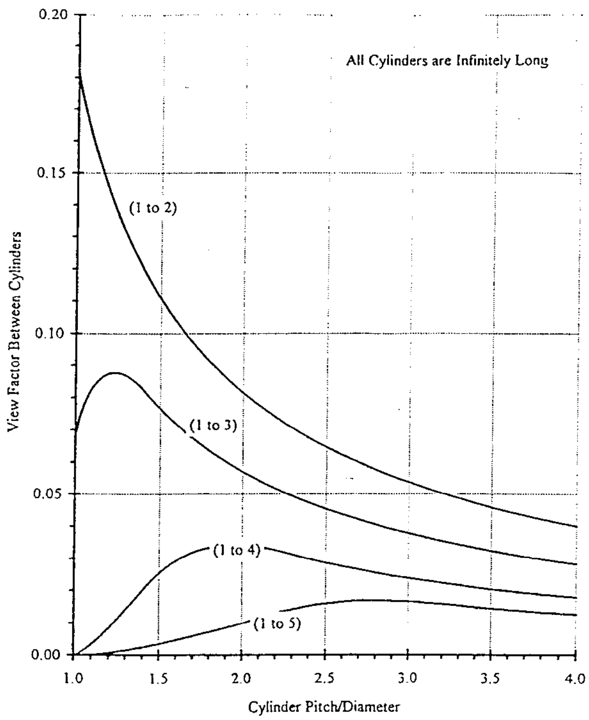

FIGURE 6. VIEW FACTORS BETWEEN INFINITE CYLINDERS AND THEIR NEIGHBORS

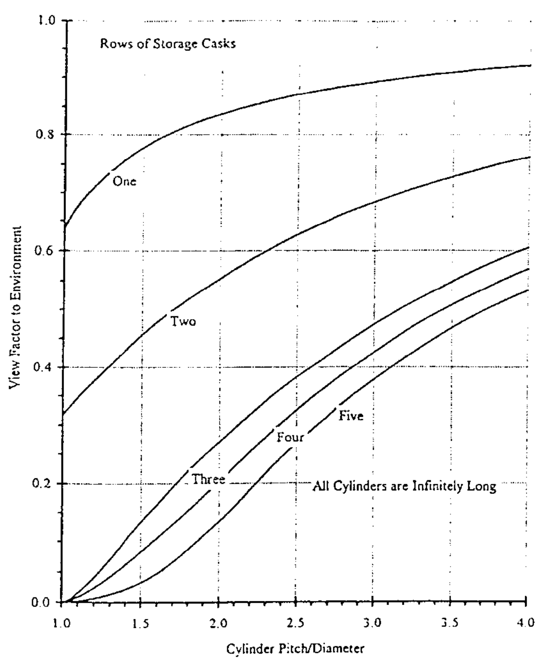

FIGURE 7. MINIMUM VIEW FACTORS BETWEEN INFINITE CYLINDERS AND THE ENVIRONMENT

A spent fuel cask represented as an infinite cylinder results in an underestimate of the view factor between the cask and the environment because the portion of the environment above and below the casks has been neglected. Consequently, the surface temperature of the cask will be overestimated.

3. VIEW FACTORS FOR FINITE-LENGTH CYLINDERS

In the previous analysis, the omission of the radiant heat transfer from a cylinder (spent fuel storage cask) to the environment above or below the cylinder could be significant. Consequently, it is necessary to also evaluate the view factors for finite-length cylinders. This calculation can be performed using MONT3D (Maltby 1993), a Monte Carlo-based computer program to determine the fraction of radiant energy that leaves one surface and reaches another surface.

The primary purpose of MONT3D (there is also a 2D version, MONT2D) is to generate realistic radiative exchange factors between any two surfaces in an arbitrary geometry for both specular and diffuse scattering of radiant energy. The exchange factor for a given geometry is generated by assigning a specular or diffuse absorptivity to each surface. In the case of a mirror, a near zero specular absorptivity is assigned. For most common materials, diffuse absorbtivities between 0 and 1 are assigned as appropriate to the material. To obtain only view factors, an absorptivity of essentially unity is assigned to all materials.

One restriction within MONT3D is that it must deal with arbitrary combinations of flat surfaces. Thus, it was necessary to represent the cylinders as a series of flat plates arranged to approximate the geometry of a cylinder. A series of preliminary calculations indicated that the representation of the cylinders chosen for this analysis presented a point of diminishing return, where increasing the resolution of the cylinders did not result in a noteworthy change in the computed view factors. In practice, the point of diminishing return for P/D=2 with three rows of casks was reached with only eight surfaces required to represent a cylinder. The geometry employed to represent portions of two rows of cylinders is presented in Figure 8. The four lateral surfaces that surround the cylinders were assigned properties of mirrors or wall surfaces as required. The mirrors are perfect reflectors to numerically represent planes of symmetry. If all of these surfaces were assigned the properties of mirrors, then they created the effect that this array was a part of an infinite array of cylinders. By combining the surface attributes as mirrors along the planes of symmetry of a finite array and the remaining surface attributes as black (perfectly absorbing), then these black surfaces would represent losses to the environment and distant members of the array (cask 6 and beyond).

MONT3D was employed to evaluate the view factor between cylinders with a P/D of 2 as a function of the length-to-diameter ratio of the cylinder. The resulting view factors are presented in Figure 9 for radiant heat transfer from cylinder 1 to each of the other cylinders of interest. Superimposed upon Figure 9 are the values from the analytic expressions for infinitely long cylinders to indicate that the two methods do approach the same value when applied to the same geometry. Assigning the appropriate surfaces the properties mirrors permits the representation of infinite-length cylinders in an infinitely long array of three rows of casks. This permits a direct comparison between the Monte Carlo and analytic solutions. As can be seen in Figure 9 and 10, in the limit of infinitely long cylinders, both approach the same value.

As with infinitely long cylinders, the view factor between a cylinder and the environment is a combination of the view factors for interchange between the individual cylinders. The expressions for the view factors between a cylinder and the environment presented in the section characterizing infinitely long cylinders have been evaluated for finite cylinders; the results are presented in Figure 10. Again, the values from the analytic expressions for infinitely long cylinders are superimposed upon Figure 10 for comparison.

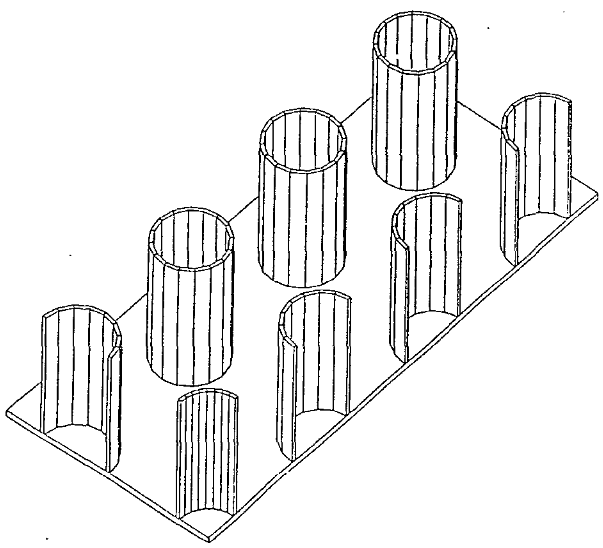

FIGURE 8. MONT3D GEOMETRY FOR EVALUATING VIEW FACTORS FOR FINITE-LENGTH CYLINDERS

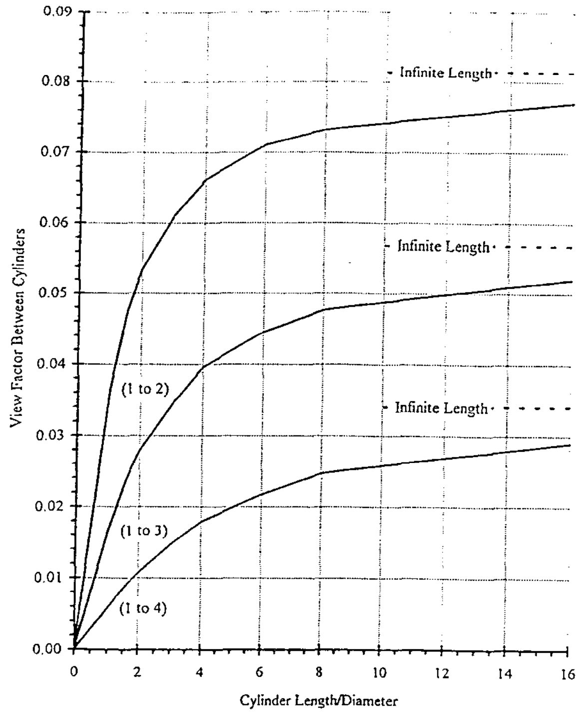

FIGURE 9. VIEW FACTORS BETWEEN CYLINDERS WITH FINITE LENGTHS

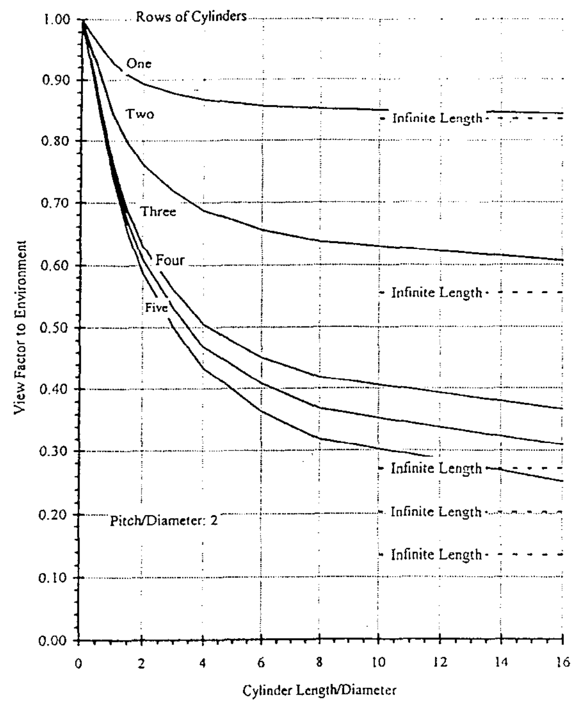

FIGURE 10. MINIMUM VIEW FACTORS BETWEEN FINITE-LENGTH CYLINDERS AND THE ENVIRONMENT

4. VIEW FACTORS FOR FINITE CYLINDERS WITH P/D ≠ 2

The view factors for radiant interchange between a cylinder (spent fuel storage cask) and the environment have different values for the two distinct approaches (infinitely long cylinder and finite length cylinder) presented in the previous sections. If P/D is exactly 2, the data in Figure 10 should be used for the view factor between a cylinder and the environment.

However, if the P/D is not 2, the following procedure should be used:

1. From Figure 7, determine the ratio of the view factor at the desired P/D to the view factor at a P/D of 2.
2. Multiply this ratio by the view factor from Figure 10 which is evaluated at the desired diameter-to-length ratio. This is the desired view factor.

In equation form this is expressed as:

$$F_{1-e}\left((L/D),\ (P/D)\right) =$$
$$\left[F_{1-e}\left((L/D),\ (P/D)=2\right)\right]\left[\frac{F_{1-e}\left((L/D)=\infty,\ (P/D)\right)}{F_{1-e}\left((L/D)=\infty,\ (P/D)=2\right)}\right].$$

This approach is valid if the P/D is close to 2. If the P/D is significantly different from 2, this approach to determining the view factor is inappropriate and the computer program MONT3D should be used for the specific case of interest.

5. REFERENCES

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